BASIC INFORMATION ON SUB-PROJECT

NAME OF PROGRAMME/FUND	Scholarship Fund - Sciex NMS^{ch}
RESEARCH FIELD AND OTHER RESEARCH FIELDS INVOLVED (if applicable)	Mathematics
TITLE OF THE SUB-PROJECT	Transport phenomena in continuum fluid dynamics (TraFlu)
REGION OF THE CZECH REPUBLIC (according to the location of the home institution)	Prague
GRANT AMOUNT SPENT	95 338,57 CHF
INTERMEDIATE BODY	Swissuniversities
HOME INSTITUTION	Academy of Sciences of the Czech Republic Institute of Mathematics
HOST INSTITUTION	University of Zürich Institut für Mathematik
NAME OF THE FELLOW	Ondřej Kreml

ABSTRACT OF THE SUB-PROJECT

A simple model of mass transport is provided by the continuity equation

$$\partial r/\partial t + \mathrm{div}(ru) = 0 \qquad (1)$$

where r stands for the mass density and u is the velocity field. In simple models like that of traffic flow, the velocity itself is a function of the density r; hence (1) reduces to a scalar conservation law. In fluid dynamics, the velocity field is coupled to u and other fields via the balance of momentum and/or other equations. Writing $\partial r/\partial t + \mathrm{div}(ru) = \partial r/\partial t + u \bullet r + r\,\mathrm{div}(u)$ we observe that first two terms represent the motion along the so-called characteristic field - solutions of the system of ordinary differential equations

$$dX(t)/dt = u(t, X(t)) \qquad (2)$$

A principal difficulty arises then from the fact that in many applications the velocity field u does not, or is not known to, possess sufficient regularity for X to be uniquely determined in terms of the initial data.

In their seminal work [11], DiPerna and Lions proposed a new approach to (2) via solving the associated transport equation (1). The crucial point of the theory is, however, boundedness of div(u) that provides uniform a priori bounds on the density in terms of the initial data. DiPerna and Lions introduced also the concept of renormalized solution to equation (1), exploited later by Lions [12] in his existence theory for the barotropic Navier-Stokes system.

Another important step in the development of the theory was done by Ambrosio [1] extending the DiPerna-Lions theory to BV velocity fields, with div(u) belonging to L^1_{loc} (see also [2] and [4]). Despite the generality of this result, some applications in particular in the theory of conservation laws and their linearizations require mere one sided Lipschitz condition to be imposed on u, in particular, div(u) may be bounded from above but not, in general, integrable, see [7].

MAIN RESULTS	Chiodaroli, E.; De Lellis, C.; Kreml, O.: Global ill-posedness of the isentropic system of gas dynamics. Comm. Pure Appl. Math. 68 (2015), no. 7, 1157–1190. Chiodaroli, E.; Feireisl, E.; Kreml, O.: On the weak solutions to the equations of a compressible heat conducting gas. Ann. Inst. H. Poincaré Anal. Non Linéaire 32 (2015), no. 1, 225–243. Chiodaroli, E.; Kreml, O.: On the Energy Dissipation Rate of Solutions to the Compressible Isentropic Euler System. Arch. Rational Mech. Anal. 214 (2014), 1019–1049.
DATE OF REALISATION OF THE FELLOWSHIP	1.10.2012 - 31.12.2013 (with break 1.2.2013 – 30.4.2013)
MORE INFORMATION ON THE PROGRAMME	**www.sciex.ch**