BASIC INFORMATION ON SUB-PROJECT

NAME OF PROGRAMME/FUND	Scholarship Fund - Sciex NMS^{ch}
RESEARCH FIELD AND OTHER RESEARCH FIELDS INVOLVED (if applicable)	Mathematics / Natural Sciences Engineering Sciences
TITLE OF THE SUB-PROJECT	Preconditioned Krylov subspace methods for large-scale model reduction (KryMoR)
REGION OF THE CZECH REPUBLIC (according to the location of the home institution)	Liberec Region
GRANT AMOUNT SPENT	95 137,53 CHF
INTERMEDIATE BODY	Swissuniversities
HOME INSTITUTION	Technical University of Liberec, Faculty of Mechatronics Institute of novel technologies and applied informatics
HOST INSTITUTION	ETH Zurich Department of Mathematics
NAME OF THE FELLOW	Martin Plešinger

ABSTRACT OF THE SUB-PROJECT	A typical use of numerical simulations is the measurement and control of output quantities such as heat, noise, and stress at critical parts of the computational domain with respect to a selected set of input parameters. The fundamental idea of *mathematical model reduction* is that this input-output behaviour can often be well approximated by a much simpler model than needed for describing the entire state of the simulation. Once model reduction has been performed, the original model can be replaced by the resulting simpler model, leading to reduced simulation times and greatly facilitating the further analysis and design of a control system. For instance, often only a low-order model allows for the use of more sophisticated robust and optimal kontrol techniques. With the advances of modern control theory, model reduction has become an important and rapidly changing field with a large diversity of application areas, including structural and fluid dynamics, biosystems, and micro-electro-mechanical systems. Among the existing techniques for model reduction, balanced truncation is considered to be very robust as it admits explicit error bounds and preserves stability. The goal of this project consists of the development and analysis of novel preconditioned Krylov subspace methods that retain the reliability of balanced truncation at a much lower computational cost compared to exisiting algorithms. In the long run, these algorithms may have a significant impact on the applicability of model reduction tomore complex models.

MAIN RESULTS	The conjugate gradient (CG) method [1] is one of the most popular methods for solving large-scale linear systems $Mx=b$ with a symmetric positive definite (SPD) matrix M. In our work, we have investigated the use of the CG method for solving the so called Lyapunov equation $AX+XA^T = K$, where A, X, K are square matrices of the same dimensions and A^T denotes the transpose of a matrix. Such equations play an important role in dynamic systems and control theory. In applications, we typically have $K = BB^T$ with a matrix B containing only a few columns [2]. For simplicity, we moreover assume that A is symmetric positive definite. Using Kronecker products, the Lyapunov equation $AX+XA^T = K$ can be rewritten as a standard linear system of the form $Mx = b$ with $M = \mathrm{kron}(A,I)+\mathrm{kron}(I,A)$, $x = \mathrm{vec}(X)$, and $b = \mathrm{vec}(K)$, where kron denotes the Kronecker product of two matrices and vec stacks the columns of a matrix into a long vector. Under the assumptions stated above, M is an SPD matrix and therefore CG is applicable, at least in principle. For a large matrix size n, however, the CG method quickly becomes impracticable, simply because of the need for storing vectors of length $n*n$. Problems appearing in practice often feature n in the order of millions or billions. Based on the ideas and algorithms in [3], we have developed a low-rank variant of the CG method that avoids this problem. The fundamental idea behind our variant is that the solution X of the Lyapunov equation can be shown to have an exponential decay in its singular values. Hence, X can be approximated very accurately by a low-rank matrix requiring significantly less storage. Our variant of the CG method approaches this low-rank approximation via low-rank iterates, implying that the storage requirements grow linearly or logarithmic-linearly (instead of quadratically) as n increases. We have shown that the CG method as well as our low-rank variant preserve the symmetry of the solution. However, it remains a difficult open problem to also control the decay of singular values (and hence the low-rank approximation quality) of the intermediate iterates. We have strong numerical evidence for such a strong decay, provided that the CG method converges sufficiently quickly. To achieve quick convergence, the use of very

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